

End Semester/Reappear (Semester I) Examination December 2024

Programme: B.Tech (MiE/ CSE)

Full Marks: 50

Course: Mathematics I

Course Code: 8BSC102/ 3BSC102

Time: 3 Hrs.

Enrollment No: _____

Section I

1. Short Answer type questions.

4 x 5 = 20

- a. Find the constant a and b such that the matrix $A = \begin{bmatrix} a & 4 \\ 1 & b \end{bmatrix}$ has 3 and -2 as its eigen value.

Understand (CO1)

Find rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ -1 & -2 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

Understand (CO1)

- b. State Rolles theorem.

Remember (CO2)

or

Evaluate $\int_0^1 x^2(1-x^2)^{\frac{7}{2}} dx$

Understand (CO2)

- c. If $u = (3xy - y^3) - (y^2 - 2x)^{\frac{3}{2}}$ determine $\frac{\partial^2 u}{\partial x \partial y}$

Understand (CO3)

or

If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$

Understand (CO3)

- d. Find the value $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dx dy}{1+x^2+y^2}$

Understand (CO 4)

or

Find the value $\int_0^2 \int_1^z \int_0^{yz} xyz dx dy dz$

Understand (CO 4)

Section II

Long Answer type questions.

3 x 10 = 30

2. Verify that the matrix $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ satisfies its own characteristic equation and hence find A^4 .

Evaluate (CO1)

or

Find eigen value and eigen vector of $\begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$.

Evaluate (CO1)

3. Examine the function $u = x^3y^2(12 - 3x - 4y)$ for extreme values. Evaluate (CO3)

or

Show that the rectangular solid of maximum value that can be inscribed in a sphere is a cube.

Evaluate (CO3)

4. Evaluate $\int \int y dx dy$ over the region enclosed by the parabola $x^2 = y$ and the line $y = x + 2$.

Evaluate (CO 4)

or

Change the order of integration of $\int_0^1 \int_x^{\sqrt{x}} f(x, y) dy dx$.

Evaluate (CO 4)

Section III

Application based questions.

1 x 20 = 20

5. Prove that if $0 < a < 1, 0 < b < 1$; then $\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1}b - \sin^{-1}a < \frac{b-a}{\sqrt{1-b^2}}$ hence deduce that

$$\frac{\pi}{6} - \frac{1}{2\sqrt{3}} < \sin^{-1} \frac{1}{4} < \frac{\pi}{6} - \frac{1}{\sqrt{15}}$$

Evaluate (CO2)

or

Evaluate (i) $\lim_{x \rightarrow 0} \frac{x^{1/2} \tan x}{(e^x - 1)^{3/2}}$ (ii) $\lim_{\theta \rightarrow \alpha} \frac{1 - \cos(\theta - \alpha)}{(\sin \theta - \sin \alpha)^2}$

Evaluate (CO2)

Course Outcome:

On the completion of the Course, the students will be able to:

CO 1: Able to calculate rank of matrix, characteristic equation & characteristic roots & use the applicability of Cayley Hamilton Theorem to find inverse of matrix which is very important in many engineering application

CO 2: Ability to understand calculus and its application in engineering

CO 3: Gain knowledge about multiple differentiations which is helpful in Engineering & it is also useful in Research & Development

CO 4: Gain knowledge about multiple Integration which is helpful in Engineering & it is also useful in Research & Development